On the Error Analysis of Single-Channel Free-Access Collision Resolution Algorithms

Robert M. Liang
Jet Propulsion Laboratory
Pasadena, CA 91109

Harry H. Tan
Electrical and Computer
Engineering Department
University of California, Irvine
Irvine, California 92717

Abstract

Future wireless ATM networks proposed for providing OC-3 and above data rates generally employ demand assigned multiple-access protocols over time division multiplexed channels. Collision resolution algorithm based random access protocols (RAP) are useful for implementing the connection request procedure for these protocols. In the wireless networking environment, it is important to determine the performance of these protocols in the presence of propagation channel errors. This paper considers the performance of a single channel Q-array free-access collision resolution algorithm based RAP in the presence of memoryless feedback channel errors. The maximum stable throughput or capacity is derived for an infinite population Poisson user population model. The average delay-throughput characteristic is also derived. Simulations were performed to validate the analytical resultes.

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1 INTRODUCTION

The internet has generated an increasing demand for high-speed multimedia services in both business and consumer telecommunication networks. Moreover, industry forecasts indicate that both satellite and ground based wireless transmission systems will become increasingly more important for providing portability and mobility capabilities as well the implementation of fixed last mile connections in these high speed broadband multimedia networks [1]. Since asynchronous transfer mode (ATM) has generally been adopted as the preferred switching and transport protocol in high speed wired networks for multimedia services, its extension to the wireless environment has received an increasing amount of recent attention [2]. Architectures of wireless ATM networks capable of supporting up to OC-3 (155 Mbps) data transmission rates have been proposed in recent research publications targeted at creating end-to-end ATM services.

Among the technological issues that have to be resolved in order to arrive at a wireless ATM network is the development of suitable multiple-access (MAC) protocols in the data link layer. Proposals for the wireless ATM networks have generally been either spread spectrum physical layer (PHY) where the natural MAC protocol is code-division multiple-access (CDMA); or a time-division multiplexed PHY with a demand-assigned MAC protocol [3]. Even through spread spectrum techniques employing CDMA have been shown to be appropriate for digital cellular networks, including the third generation IMT-2000 system, it has a severe disadvantage for high data transmission rates because of the large bandwidths required to support an adequate spread ratio. Therefore proposals for future wireless ATM networks targeted at providing OC-3 data rates generally employ demand-assigned MAC protocols over time-division multiplexed channels. Many of those proposed networks use a random-access protocol for the dial-up or connection request procedure in the demand-assigned MAC protocol.

Random access protocols (RAP) are useful for implementing the connection request procedure in demand-assigned multiple access systems. The Q-ary single-channel algorithms can be used in the multiple channel time division multiplexed environment. Each user with a new packet would choose one of the Q channels to use with probability $1/Q$ and continue on that channel until that packet is successfully transmitted. In a RAP, a collision is said to occur whenever two or more transmitters attempt to transmit a packet over the same time slot. All packets involved in a collision are assumed to be destroyed and have to be retransmitted. Collision resolution algorithms (CRA) can be used to organize the retransmission times of collided packets.

The single-channel binary feedback window blocked-access CRA based RAPs [5], [6] achieved a maximum stable channel throughput of .4294. Ternary feedback protocols [7] , [9] provide even higher maximum stable throughput with the highest achievable throughput to date of .48775 [10]. Ternary feedback blocked-access CRA based RAPs however have the undesirable drawback of catastrophic deadlock failures in the presence of channel feedback errors. In spite of their lower maximum stable throughputs, there has also been substantial interest in single-channel free-access CRA based RAPs [8], [11], [12], [13] because of their implementation advantage over window blocked-access algorithms. Mathys and Flajolet [13] has obtained the best single-channel free-access CRA based RAPs to date with stable throughputs of .4016 and .4076 in the binary and ternary feedback situations.

Wang and Thanawastien [14] have designed a Q-channel window blocked-access algorithm which resolve collisions on the channel by channel basis. That is if a collision occurs in one of the channels, than all Q-channels are used to resolve the collision. However, this approach is inefficient for large Q (i.e., $Q \geq 4$). This is intuitively evident from the fact that, for large Q, many empty channel slots are created and did not utilized. The multiple-channel free-access (MCFA) algorithm [4] exploit those empty channel slots by transmitting new arrivals.

In the wireless network environment, the effect of propagation channel errors on the performance of the multiple-access protocol is important. Both memoryless channel model and the Gilbert channel model have received enormous amount of attention in the literature for the studies of block-access or splitting algorithms in the presence of channel errors. The system capacity of single channel block access algorithms in the presence of memoryless channel errors were considered in [9] and [17]-[19]. The system capacity of single channel block access algorithms in the presence of Gilbert channel errors were considered in [20]. However, there appears to be no previous work on the performance of the free-access algorithm in the presence of channel errors. Performance analysis fo the Q-ary single channel free-access collision resolution algorithm in the presence of feedback channel errors will provide valuable insight into the performance of demand assigned multiple access protocols operating in the wireless environment.

In this paper, we derive the performance of Q-ary single channel free-access collision resolution algorithm. In Section 2, we give a brief description of the Q-ary single channel free-access algorithm. Throughput and delay analysis are given in Sections 3 and 4, respectively. Conclusions and discussions are presented in Section 5.

2 THE BASIC Q-ARY SCFA

In this section, we provide some brief background of the binary feedback Q-ary single-channel free-access collision resolution algorithm. The Q-ary single-channel free-access algorithm is summarized as follows: The new packets are transmitted immediately at the beginning of the next slot following their arrival (regardless of whether there is any collision resolution in progress or not). After a collision, each user, involved in the conflict, chooses one of Q-groups for transmission with equal probability. This splits the set of contending packets into Q subsets or groups. Those who select group number one transmit first and follow by group number two and etc. The second group is transmitted only when all packets are resolved in the first group.

Implementation of the above protocol at each transmitter (user) is based on the counter scheme. Each transmitter is assumed at any given time to have at most one packet either ready of in the process of transmission. The packet transmission time is indicated by a counter kept by each transmitter. Packet transmission are initiated only when the value of the counter is zero. The value of the counter is either incremented or decremented based on the status of the feedback information.

3 CAPACITY IN THE PRESENCE OF FEEDBACK ERRORS

In this section, we examine the maximum achievable stable throughput of the binary feedback Q-ary free-access collision resolution algorithm in the presence of feedback errors under the assumption of an infinite user population. That is the new packet arrival point process which is assumed to be Poisson with rate λ packets per slot. We assume a memoryless feedback channel error model such that collision state feedback has no error and the no collision state feedback has an error with probability ϵ. Feedback information errors cause idle and single packet channel slots to be interpreted as collision slots. Extra time slots are required to resolve these erroneous collision slots. Errors are assumed to be independent from slot to slot and independent of the packet arrival process.

The throughput analysis, presented here, is based on some of the techniques first developed by [9] for analysis the effect of channel errors for the binary obvious blocked-access algorithm along with the generating function techniques previously applied by [13] for analyzing single channel free-access algorithm (without feedback errors). The collision resolution interval (CRI) is defined as the time period from the start of the slot where the initial collision occurs up to and including the slot when the initial collision has been resolved. The CRIs play a fundamental role in this analysis. Since CRIs can be nested, a CRI which is part of a larger CRI is called a sub-CRI; and a CRI which is not a sub CRI of any other CRI is called a CRI from scratch.

For a generic CRI from scratch, let U denote the number of packets transmitting in the present slot and let

Y(U) denote the length of the CRI measured in slots in the error-free environment. Moreover, define the following: Y_b is the number of blank slots in the CRI; Y_s is the number of slots with a single packet in the CRI; and Y_c is the number of the slots with collision in the CRI. In the error-free case, it is observed that the single channel Q-ary algorithm is essentially a Q-ary tree scheme where the number of intermediate nodes of the Q-ary tree is Y_c and the number of terminal nodes is $Y_s + Y_b$. By starting with a full grown tree and pruning branch by branch and using mathematical induction, the following relationship can be easily established in error-free case

$$Y_b(U) + Y_s(U) = (Q-1)Y_c(U) + 1. \tag{1}$$

Note that for binary collision resolution algorithm

$$Y_b(U) + Y_s(U) = Y_c(U) + 1$$

which was first derived by [9]. Furthermore,

$$Y(U) = Y_c(U) + Y_s(U) + Y_b(U). \tag{2}$$

For the error-free environment, define the conditional expected values

$$\begin{aligned} L_N &= E\{Y(U)|\text{U=N, in error free environment}\} \\ S_N &= E\{Y_s(U)|\text{U=N, in error free environment}\} \\ B_N &= E\{Y_b(U)|\text{U=N, in error free environment}\} \\ C_N &= E\{Y_c(U)|\text{U=N, in error free environment}\} \end{aligned}$$

Given that $U = N$, the expected number of new packet arriving and successfully transmit in the CRI is equal to $\lambda(L_N - 1)$. Hence,

$$S_N = N + \lambda(L_N - 1). \tag{3}$$

Using (3) it follows from taking conditional expectations (1) and (2) that

$$L_N = N + \lambda(L_N - 1) + B_N + C_N, \tag{4}$$

$$B_N + N + \lambda(L_N - 1) = (Q-1)C_N + 1, \tag{5}$$

which obtains

$$B_N = L_N(1 - 1/Q - \lambda) + 1/Q + \lambda - N. \tag{6}$$

In the environment with feedback channel errors, define the conditional CRI length as

$$L_{N,e} = E\{Y(U)|\text{U = N, in channel error environment}\}. \tag{7}$$

Define T_b to be the number of time slots required for the successful resolution of a blank channel slot when feedback channel error are present. T_s denotes the number of time slots required for the successful resolution of a single packet channel slot when feedback channel error are present. Then

$$T_b = \begin{cases} 1 & \text{with probability (1-}\epsilon) \\ 1 + QY(X) & \text{with probability } \epsilon \end{cases} \tag{8}$$

$$T_s = \begin{cases} 1 & \text{with probability (1-}\epsilon) \\ 1 + (Q-1)Y(X) + Y(X+1) & \\ & \text{with probability } \epsilon \end{cases} \tag{9}$$

where X is a Poisson random variable with mean λ representing the new arrival packets transmitting in the first slot of the sub-CRI's that resolves the blank channel slot or the single packet channel slot. It follows from equations (8) and (9) that

$$\begin{aligned} E\{T_b\} &= (1-\epsilon) + \epsilon(1 + \\ &\quad Q\sum_{x=0}^{\infty} \frac{\lambda^x}{x!} e^{-\lambda} E\{Y(U)|U = x, error\}) \\ &= 1 + \epsilon Q \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} e^{-\lambda} L_{x,e}. \end{aligned} \tag{10}$$

Similarly,

$$E\{T_s\} = 1 + \epsilon(Q-1)\sum_{x=0}^{\infty} \frac{\lambda^x}{x!} e^{-\lambda} L_{x,e} + \epsilon \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} e^{-\lambda} L_{x+1,e}. \tag{11}$$

Similar to Messey [9], we conclude that

$$L_{N,e} = L_N + B_N E\{T_b - 1\} + S_N E\{T_s - 1\}. \tag{12}$$

where $E\{T_b - 1\}$ is the expected number of extra slots, induced by feedback error, when a blank was transmitted and $E\{T_s - 1\}$ is the expected number of extra slots, induced by feedback error, when a single packet was transmitted. When $N = 0$, using (12), (3), (6), (10), and (11) can show

$$L_{0,e} = 1 + E\{T_b - 1\}. \tag{13}$$

Similarly, when $N = 1$,

$$L_{1,e} = 1 + E\{T_s - 1\}. \tag{14}$$

Define the generating function

$$\begin{aligned} L_e(z) &= \sum_{N=0}^{\infty} L_{N,e} \frac{z^N}{N!} \\ L_e^*(z) &= e^{-z} L_e(z) \end{aligned} \tag{15}$$

Then it follows from (10), (11), (13), (14), and (15) that it can be established that

$$L_e^*(0) = L_{0,e} = 1 + \epsilon Q L_e^*(\lambda) \tag{16}$$

and

$$L_e^{*(1)}(0) = \epsilon L_e^{*(1)}(\lambda). \tag{17}$$

By taking the generating function of (12), $L_e^*(z)$ can be immediately obtained in terms of $L^*(z)$ in the error-free case. That is

$$\begin{aligned} L_e^*(z) &= L^*(z)\{1 + (Q-1)\epsilon L_e^*(\lambda) + \lambda\epsilon L_e^{*(1)}(\lambda)\} \\ &\quad + \epsilon L_e^*(\lambda) + (z-\lambda)\epsilon L_e^{*(1)}(\lambda). \end{aligned} \tag{18}$$

Taking the derivative of (18) and setting $z = \lambda$, $L_e^{*(1)}(\lambda)$ can be solved. Substituting $L_e^{*(1)}(\lambda)$ back into (18) and setting $z = \lambda$, an expression for $L_e^*(\lambda)$ can be obtained in terms of $L^*(\lambda)$ and $L^{*(1)}(\lambda)$ in the error-free environment. However, this intuitive approach does not provide us sufficient amount informations regarding the behavior of $L_e^*(z)$ which are required when we compute the delay-throughput characteristics. We need to derive a functional equation of $L_e^*(z)$ which solely governed by $L_e^*(\lambda)$ and $L_e^{*(1)}(\lambda)$.

Let $Y_e(U)$ denote the length of CRI, measured in slots when feedback errors are presented. The U packets initiating the CRI independently select among the Q groups for transmission. Let U_j denote the number of packets selecting j-th group for transmission where $j \leq Q$. Clearly, $\sum_{j=1}^{Q} U_j = U$. A collision occurs in the j-th group if and only if the number of packets in the j-th group is greater than two. So the collision (if any) in the first group is first resolved, followed by the collision (if any) in the second group, and so on. Resolution of the collision (if any) in the j-th group starts a sub-CRI. These sub-CRIs evolve disjointly in time. Let X_j denote the newly arrived packets during the first slot of the sub-CRI. Since the sub-CRIs are disjoint in time, $X_1, X_2, \ldots, X_Q$ are i.i.d. Poisson random variables with mean λ that also are independent of $U_1, U_2, \ldots, U_Q$. Since the evolution of each of these sub-CRIs is similar to the evolution of a CRI from scratch, the length of the sub-CRI resolving the set of U_j packets is denoted by $Y_e(U_j + X_j)$. Note that the conditional distribution of $Y_e(U)$ is identical to the conditional distribution of $Y_e(U_j + X_j)$, given that $U = U_j + X_j = N$. Hence,

$$Y_e(U) = \sum_{j=1}^{Q} Y_e(U_j + X_j) \tag{19}$$

It can be shown that for $N \geq 2$

$$L_{N,e} = 1 + Q \sum_{i=0}^{N} \binom{N}{i} \left(\frac{1}{Q}\right)^i \left(1 - \frac{1}{Q}\right)^{N-i} e^{-\lambda} L_e^{(i)}(\lambda). \tag{20}$$

Taking the generating function of L_N, e,

$$L_e(\lambda) = L_{0,e} + zL_{1,e} + \sum_{N=2}^{\infty} L_{N,e} \frac{z^N}{N!}. \tag{21}$$

Substituting (13), (14), and (20) into (21),

$$\begin{aligned} L_e(z) &= E\{T_b - 1\} + zE\{T_s - 1\} + e^z \\ &\quad + Qe^{-\lambda} \sum_{N=2}^{\infty} \frac{z^N}{N!} \sum_{i=0}^{N} \binom{N}{i} \\ &\quad \left(\frac{1}{Q}\right)^i \left(1 - \frac{1}{Q}\right)^{N-i} L_e^{(i)}(\lambda). \end{aligned} \tag{22}$$

Hence,

$$\begin{aligned} L_e^*(z) &= 1 + e^{-z}E\{T_b - 1\} + ze^{-z}E\{T_s - 1\} \\ &\quad + Qe^{z-\lambda} \sum_{N=2}^{\infty} \frac{z^N}{N!} \sum_{i=0}^{N} \binom{N}{i} \\ &\quad \left(\frac{1}{Q}\right)^i \left(1 - \frac{1}{Q}\right)^{N-i} L_e^{(i)}(\lambda). \end{aligned} \tag{23}$$

After some simplification,

$$\begin{aligned} L_e^*(z) &= 1 + e^{-z}E\{T_b - 1\} + ze^{-z}E\{T_s - 1\} \\ &\quad QL_e^*(\lambda + z/Q) - Q(1+z)e^{-z}L_e^*(\lambda) \\ &\quad -ze^{-z}L_e^{*(1)}(\lambda) \end{aligned} \tag{24}$$

Using (10), (11), and (24) can be written as

$$L_e^*(z) - QL_e^*(\lambda + \frac{z}{Q}) = 1 + QL_e^*(\lambda)f(z) + L_e^{*(1)}(\lambda)g(z) \tag{25}$$

where

$$\begin{aligned} f(z) &= -(1-\epsilon)(1+z)e^{-z} \\ g(z) &= -(1-\epsilon)ze^{-z} \end{aligned} \tag{26}$$

with initial conditions

$$\begin{aligned} L_e^*(0) &= 1 + Q\epsilon L_e^*(\lambda) \\ L_e^{*(1)}(0) &= \epsilon L_e^{*(1)}(\lambda). \end{aligned} \tag{27}$$

The form of the functional equation of $L_e^*(z)$ in the presence of feedback error is similar to the error-free case. As ϵ approaches zero, the function equation of $L_e^*(z)$ becomes the functional equation in error-free case with the initial conditions $L_e^*(0) = 1$ and $L_e^{*(1)}(0) = 0$ as in [13]. The solution of (25) can be obtained by using the iterative approach, suggested by Mathys and Flajolet [13]. Define for any function h(z)

$$\begin{aligned} R^{(1)}(h,z) &= \sum_{m=0}^{\infty} h^{(1)}(\lambda_m + zQ^{-m}) - h^{(1)}(\lambda_m) \\ &= h^{(1)}(Q\mu) - h^{(1)}(0) \\ R(h,z) &= \sum_{m=0}^{\infty} Q^m\{h(\lambda_m + zQ^{-m}) \\ &\quad -h(\lambda_m) - zQ^{-m}h^{(1)}(\lambda_m)\} \end{aligned} \tag{28}$$

where

$$\lambda_m = \begin{cases} 0 & \text{m=0} \\ \lambda\left(\frac{1-Q^{-m}}{1-Q^{-1}}\right) & \text{for } m \geq 1. \end{cases} \tag{29}$$

and

$$\mu = \frac{\lambda}{1 - Q^{-1}} \tag{30}$$

Taking the second derivative of (25),

$$\begin{aligned} L_e^{*(2)}(z) - \frac{1}{Q}L_e^{*(2)}(\lambda + z/Q) &= Qf^{(2)}(z)L_e^*(\lambda) \\ &\quad + g^{(2)}(z)L_e^{*(1)}(\lambda) \end{aligned} \tag{31}$$

Using the iterative solution and integrating the above function,

$$\begin{aligned} L_e^{*(1)}(z) - \epsilon L_e^{*(1)}(\lambda) &= QR^{(1)}(f;z)L_e^*(\lambda) \\ &\quad + R^{(1)}(g;z)L_e^{*(1)}(\lambda) \end{aligned} \tag{32}$$

Setting $z = \lambda$

$$L_e^{*(1)}(\lambda) = \frac{QL_e^*(\lambda)R^{(1)}(f;\lambda)}{(1-\epsilon) - R^{(1)}(g;\lambda)} = \frac{Q\mu}{1-\mu}L_e^*(\lambda). \quad (33)$$

Finally, integrating (32) once more, using (33) and setting $z = \lambda$

$$L_e^*(\lambda) = \frac{1}{1 - Q\left(\epsilon + R(f;\lambda) + \frac{R^{(1)}(f;\lambda)\{\lambda\epsilon + R(g;\lambda)\}}{(1-\epsilon)-R^{(1)}(g;\lambda)}\right)} \quad (34)$$

where $R^{(1)}(f;\lambda)$, $R^{(1)}(g;\lambda)$, $R(f;\lambda)$, and $R(g;\lambda)$ are given in the Appendix I. The maximum stable throughput, λ_{max}, of the Q-ary single channel free-access algorithm in presence of feedback channel error is the argument which maximizes (34).

4 DELAY ANALYSIS IN THE PRESENCE OF FEEDBACK ERRORS

The delay of a packet will be measured from the beginning of its first transmission to the beginning of the slot in which it is successfully transmitted. This may be referred to as the packet session delay, and is equal to an integer number of slots. The total delay incurred by a packet is the session delay plus one slot for the packet transmission time and plus the time difference between the packet arrival time and the beginning of the next slot following that arrival time. Hence the total average delay is equal to the average session delay plus 1.5 slots in the zero propagation delay situation.

Similar to the approach taken in [15] and [16], the average packet session delay will be determined using the following law of large numbers (LLN) consideration. Specifically, the average session delay is obtained by taking the sum of the random session delays of all packets which are successfully transmitted over a sequence of M CRI's from scratch, divided by the random number of such packets, in the limit as $M \to \infty$. The LLN applies, since the sequence of M CRI's from scratch evolve in a statistically independent manner due to the Poisson new packet arrival process. Note from the LLN that the number of packets successfully transmitted in the sequence of M CRI's is, in the limit as M tends to ∞, almost surely equal to $\lambda \cdot M \cdot L^*(\lambda)$.

For the CRI from scratch with U packets transmitting in the first slot, let $W_e(U)$ denote the random delay of a packet, $\mathbf{x}$, whose first time transmission results a collision with multiplicity U and the channel has error probability ϵ. If the collision has occurred, each of the U packets in the first slot of the CRI from scratch independently chooses one of the Q groups to transmit with equal probability $1/Q$. We refer this process as the slitting stage. Let U_j denote the number of packets selecting group j for transmission and let X_j denote the newly arrived packets

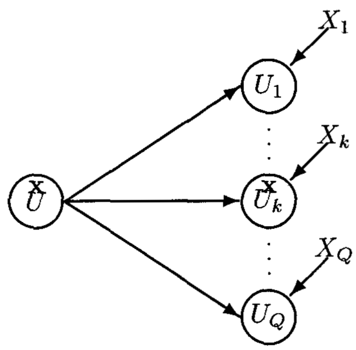

Figure 1: A Q-ary tree diagram. U is the number of the packets that initiated collision and the channel has error probability ϵ. The $\mathbf{x}$ is one of the U packets.

transmitted during the first slot of the sub-CRI (if any) resolving the set of U_j packets, for $1 \le j \le Q$. After the splitting stage, let the packet $\mathbf{x}$ be one of U_k packets as illustrated in Figure 1. The delay of the packet, $\mathbf{x}$, is one plus the sum of the CRI's of $U_1+X_1, \cdots, U_{k-1}+X_{k-1}$ and plus the amount of time required to transmit the packet, $\mathbf{x}$, with initial collision of multiplicity $U_k + X_k$. For instance, if the packet $\mathbf{x}$ in Figure 1 is one of the packets in U_3, then the random delay of the packet $\mathbf{x}$ whose first time transmission results in conflict with multiplicity U and the channel has error probability ϵ is

$$W_e(U) = 1 + Y_e(U_1+X_1) + Y_e(U_2+X_2) + W_e(U_3+X_3) \quad (35)$$

where $Y_e(U_1+X_1)$ and $Y_e(U_2+X_2)$ are the length of the CRI's of U_1+X_1 and U_2+X_2, respectively. Moreover, noting that the conditional distribution of $W_e(U)$ is identical to that of $W_e(U_j+X_j)$, given that $U = U_j+X_j = N$. Since each of the U packets in the first slot of the CRI from scratch independently chooses one of the Q groups for transmission with equal probability $1/Q$, the following recursion in U valid for $U \ge 2$

$$W_e(U) = \begin{cases} 1 + W_e(U_1+X_1) \\ \qquad\qquad \text{with probability } \frac{U_1}{U}, \\ 1 + Y_e(U_1+X_2) + W_e(U_2+X_2) \\ \qquad\qquad \text{with probability } \frac{U_2}{U}, \\ 1 + \sum_{j=1}^{k-1} Y_e(U_j+X_j) + W_e(U_k+X_k) \\ \qquad\qquad \text{with probability } \frac{U_k}{U} \\ \qquad\qquad \text{for } 1 \le k \le Q. \end{cases} \quad (36)$$

Define the conditional moment generating function

$$A_{N,e}(s) = \sum_{k=0}^{\infty} Prob\{W_{N,e} = k\}s^k = \mathrm{E}[s^{W_e(U)}|U=N] \quad (37)$$

and the conditional expected value

$$D_{N,e} = \mathrm{E}[W_e(U)|U=N] = A_{N,e}^{(1)}(1) \quad (38)$$

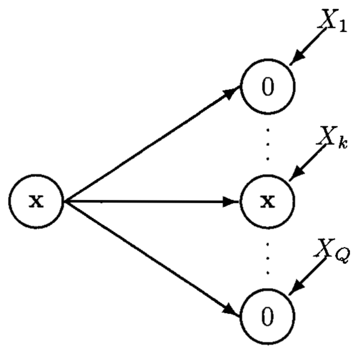

Figure 2: The single packet, **x**, selects one of Q groups for transmission with probability 1/Q and the channel has error probability ϵ.

of $W_e(U)$ given $U = N$. First, the following recursion valid for $N \geq 2$ can be obtained using (36) and (37)

$$A_N(s) = s \sum_{\substack{i_1,\dots,i_Q \\ i_1+\dots+i_Q=N}}^{N} \binom{N}{i_1,\dots,i_Q} Q^{-N} \left\{ \frac{i_1}{N} E\{s^{Y_e(i_1+X_1)}\} + \sum_{k=2}^{Q} \frac{i_k}{N} \prod_{j=1}^{k-1} E\{s^{Y_e(i_j+X_j)}\} E\{s^{W_e(i_k+X_k)}\} \right\} \quad (39)$$

Taking the derivative of (39) and setting $s = 1$, $D_{N,e}$ is obtained after some algebra for $N \geq 2$ as

$$\begin{aligned} D_{N,e} &= 1 + \sum_{i=0}^{N-1} \binom{N-1}{i} (1/Q)^i (1-1/Q)^{N-1-i} e^{-\lambda} D_e^{(i)}(\lambda) \\ &+ \frac{Q-1}{2} \sum_{i=0}^{N-1} \binom{N-1}{i} (1/Q)^i [(1-1/Q)^{N-1-i} e^{-\lambda} L_e^{(i)}(\lambda). \end{aligned} \quad (40)$$

Define next the generating functions $D_e(z)$ and $D_e^*(z)$ of the sequence $\{D_{N,e}\}$ by

$$D_e(z) = \sum_{N=0}^{\infty} D_{N+1,e} \frac{z^N}{N!} \quad (41)$$

$$D_e^*(z) = e^{-z} D_e(z). \quad (42)$$

$D_{0,e}$ is delay of a blank slot over the noisy channel. Since there is no information being transmitted, we arbitrarily set $D_{0,e}$ to be zero as in the error-free case. However, $D_{1,e} = E\{W_e(1)\}$. Hence,

$$D_{1,e} = (1-\epsilon) \cdot 0 + \epsilon E\{W_e(1)|\text{channel error}\} \quad (43)$$

$W_e(1)$ is the random delay of a single packet, **x**, and it is nonzero since the single packet, **x**, is transmitted with probability $1-\epsilon$. If the collision which induced by the feedback error has occurred, then **x** independently selects one of the Q groups for transmission with equal probability 1/Q. Suppose that **x** has selected the k-th group as shown in Figure 2. Then the delay of **x** is one plus the sum of the length of the CRI's of the first group to the (k-1)-th group plus the amount of time required to transmit **x** given the initial collision of multiplicity $1 + X_k$. Note that the lengths of the CRI's of group one to group (k-1) are equal to $E\{Y_e(X_j)\} = L_e(\lambda)$ for $1 \leq j \leq (k-1)$ where X_j denotes the newly arrived packets. Therefore, it can be established that

$$D_{1,e} = \epsilon \left\{ 1 + \frac{Q-1}{2} L_e^*(\lambda) + D_e^*(\lambda) \right\} \quad (44)$$

Taking the generating function of (40) with the initial conditions, described above, yields the following functional equation after some diligent works

$$\begin{aligned} &D_e^*(z) - D_e^*(\lambda + z/Q) + (1-\epsilon)e^{-z} D_e^*(\lambda) \\ &= [1 - (1-\epsilon)e^{-z}] \\ &+ \left(\frac{Q-1}{2}\right) \{L_e^*(\lambda + z/Q) \\ &- (1-\epsilon)e^{-z} L_e^*(\lambda)\} \end{aligned} \quad (45)$$

Let $\theta_e(U,k)$ denote the total number of packets that experience a delay k during a CRI from scratch with initial collision of multiplicity U and with channel error ϵ. $\theta_e(U,k)$ is in fact equal to U times the probability that a packet experiences a delay k. By carefully examine the splitting stage in Figure 1, we conclude that

$$\theta_e(U,k) - \sum_{j=1}^{Q} \theta_e(U_j + X_j, k) = \Delta_{U,U_j,X_j}(k) \quad (46)$$

where $\Delta_{U,U_j,X_j}(k)$ is the random increment or decrement in the number of packets that experience delay k which is induced by one splitting stage with given U, U_j, and X_j for $1 \leq j \leq Q$. Hence, given $U = N$

$$\begin{aligned} \Delta_{U=N,U_j=u_j,X_j=x_j}(k) &= N \\ Pr\{W_e(U) = k|U = N\} &- \sum_{j=1}^{Q} u_j \\ Pr\{W_e(U_j + X_j) &\mid U_j = u_j, X_j = x_j\} \end{aligned} \quad (47)$$

Note that the principle here is identical to the one in error-free case. However, the probability distribution of $W_e(U)$ is different. Let

$$V_{N,e}(k) = \mathrm{E}[\theta_e(U,k)|U = N] \quad (48)$$

Defining the generating function of $V_{N,e}(k)$ as

$$B_{N,e}(s) = \sum_{k=0}^{\infty} V_{N,e}(k) s^k. \quad (49)$$

And we denote the conditional expected total time that packets spend in the system during a CRI from scratch with initial collision of multiplicity N and with channel error ϵ as $J_{N,e}$, then

$$J_{N,e} = \sum_{k=0}^{\infty} kV_{N,e}(k) = B_{N,e}^{(1)}(1). \tag{50}$$

Using (46), (47), (48), and (49),

$$\begin{aligned} B_{N,e}(s) &= \sum_{j=1}^{Q}\sum_{i=0}^{N}\binom{N}{i}(1/Q)^i(1-1/Q)^{N-i} \\ &\quad \sum_{x=0}^{\infty}\frac{\lambda^x}{x!}e^{-\lambda}B_{x+i,e}(s) \\ &+ NA_{N,e}(s) \\ &- \frac{N}{Q}\sum_{j=1}^{Q}\sum_{i=0}^{N-1}\binom{N-1}{i}(1/Q)^i \\ &\quad (1-1/Q)^{N-1-i} \\ &\quad \sum_{x=0}^{\infty}\frac{\lambda^x}{x!}e^{-\lambda}A_{i+1+x}(s) \end{aligned} \tag{51}$$

Taking the derivative of $B_{N,e}(s)$ and setting $s = 1$, we obtain for $N \geq 2$

$$\begin{aligned} J_{N,e} &= Q\sum_{i=0}^{N}\binom{N}{i}(1/Q)^i(1-1/Q)^{N-i}e^{-\lambda}J_e^i(\lambda) \\ &+ ND_{Ne} - \sum_{i=0}^{N-1}\binom{N-1}{i}(1/Q)^i \\ &\quad (1-1/Q)^{N-1-i}e^{-\lambda}D_e^i(\lambda). \end{aligned} \tag{52}$$

Note that in error-free environment, J_0 and J_1 are zeros. However, this is not true when the system is operating in the presence of the feedback errors. $J_{0,e}$ is in fact the total amount of time that new arrivals spend in the system when the feedback error occurs. Since there are Q groups,

$$J_{0,e} = \epsilon Q J_e^*(\lambda). \tag{53}$$

To obtain $J_{1,e}$, let us carefully examine Figure 2. When the "virtual conflict" which induced by the feedback error has occurred, the packet **x** randomly selects one of Q groups to transmit. Hence, there are (Q-1) groups which each group contains only new arrival X. There is a group which contains 1+X packets, namely the packet **x** plus the new arrivals. The total time that 1+X packets spend in the system is $e^{-\lambda}J_e^{(1)}(\lambda)$. Moreover, the packet **x** has spend an additional time slot in the system. Hence,

$$\begin{aligned} J_{1,e} &= \epsilon[1 + (Q-1)J_e^*(\lambda) + J_e^{(1)}(\lambda)e^{-\lambda}] \\ &= \epsilon[1 + QJ_e^*(\lambda) + J_e^{*(1)}(\lambda)]. \end{aligned} \tag{54}$$

Define the generating functions $J_e(z)$ and $J_e^*(z)$ of the sequence $\{J_{N,e}\}$ by

$$J_e(z) = \sum_{N=0}^{\infty} J_{N,e}\frac{z^N}{N!} \tag{55}$$

$$J_e^*(z) = e^{-z}J_e(z). \tag{56}$$

Using (44), (45), (52), (53), (54), and (56) and after some algebra, we can show that

$$\begin{aligned} J_e^*(z) - QJ_e^*(\lambda + z/Q) &= Qf(z)J_e^*(\lambda) + \\ g(z)[J_e^{*(1)}(\lambda) + 1] + z + \rho(z) & \end{aligned} \tag{57}$$

where

$$\rho(z) = z\left(\frac{Q-1}{2}\right)[L_e^*(\lambda + z/Q) - e^{-z}L_e^*(\lambda)] \tag{58}$$

and $f(z)$ and $g(z)$ are defined in (26). The initial conditions for (57) are

$$J_e^*(0) = J_{0,e} = \epsilon Q J_e^*(\lambda) \tag{59}$$

and

$$J_e*(1)(0) = J_{1,e} - J_{0,e} = \epsilon + \epsilon J_e^{*(1)}(\lambda) \tag{60}$$

It can be shown that (57) satisfies the above initial conditions. The solution of $J_e^*(\lambda)$ can be obtained by using the iterative approach. It can be shown that

$$\frac{J_e^*(\lambda)}{L_e^*(\lambda)} = \frac{\lambda\epsilon + R(g;\lambda)}{(1-\epsilon) - R^{(1)}(g;z)}\left\{1 + R^{(1)}(\rho;\lambda)\right\} + R(\rho;\lambda) \tag{61}$$

Hence, the average delay is given by

$$\begin{aligned} D_{ave,e} &= 1.5 + \frac{J_e^*(\lambda)}{\lambda L_e^*(\lambda)} \\ &= 1.5 + \frac{\lambda\epsilon + R(g;\lambda)}{\lambda(1-\epsilon)(1-\mu)e^{-\mu}} \\ &\quad \left\{1 + R^{(1)}(\rho;\lambda)\right\} + \frac{R(\rho;\lambda)}{\lambda} \end{aligned} \tag{62}$$

where all parameters are specified in the Appendix I.

5 DISCUSSION AND CONCLUSION

The maximum stable throughput, λ_{max}, of the Q-ary SCFA algorithm in the presence of the memoryless feedback channel error has been determined in this paper. Numerical results for λ_{max} for various feedback error rate, ϵ, are given in Figure 3 for Q=2 to Q=7. In the error-free environment, our results are consistent with the results in [13]. Figure 3 indicates that λ_{max} decreases as the error rate ϵ increases. In fact, when $\epsilon = 1/Q$, the maximum achievable throughput is zero. To examine this situation, we define "virtual conflict" as the conflict induced by error, ϵ, when a single packet is transmitted. When a "virtual conflict" occurs, the single packet selects one of Q-groups for retransmission. It requires Q transmissions , consisting (Q-1) blanks and the single packet. When $\epsilon = 1/Q$, another error would occur with probability one. So the system is always trying to resolve errors. No information can be transmitted. Figure 3 also illustrates that the incremental reduction of capacity as ϵ increases. For example, When Q=3, the maximum achievable stable throughputs for the following values of ϵ 0, 0.05, 0.1, 0.15, and 0.2 are 0.401599, 0.378885, 0.352864, 0.322268, and 0.284831 packets per slot, respectively. As ϵ increases from 0 to 0.05, from 0.05 to 0.1, from 0.1 to 0.15, and from 0.15 to 0.2, the maximum stable throughput decreases by the amount of 0.0227, 0.0260, 0.0306, and 0.0374, respectively. When Q=4, the maximum achievable stable throughputs for ϵ equals 0, 0.05, 0.1, and 0.15 are 0.399223, 0.366924, 0.328166, and 0.278949 packets per slot, respectively. As ϵ increases from 0 to 0.05, from 0.05 to 0.1, and from 0.1 to 0.15 the maximum stable throughput decreases by the amount of 0.0323, 0.0388, and 0.0492, respectively. Therefore, the Q-ary SCFA algorithm is more sensitive to errors as the number of partitions, Q, is large.

The delay-throughput characteristics of the Q-ary SCFA algorithm in the presence of errors have been derived analytically. To examine the validity of the analytical delay analysis which was presented in the previous section, the simulated delay throughput characteristics in the presence of feedback channel errors for the Q-ary SCFA were obtained by assuming Poisson infinite-user population model. These simulations were carried out in a time duration of one million time slots. The results are shown in Figure 4. Figures 5 and 6 shows the delay-throughput characteristics of the Q-ary SCFA algorithm for various values of ϵ and for Q=3 and 4. When Q=3 and $\lambda = 0.25$, the incremental increases in average delay are 0.8328, 1.4098, 2.9048, and 10.5945 as ϵ increases from 0 to 0.05, from 0.05 to 0.1, from 0.1 to 0.15, and from 0.15 to 0.2, respectively. When Q=4 and $\lambda = 0.25$, the incremental increases in average delay are 1.2277, 2.8143, and 14.5467 as ϵ increases from 0 to 0.05, from 0.05 to 0.1, and from 0.1 to 0.15, respectively. Those figures illustrates that the average delay increases dramatically as ϵ increases for fixed Q. The best over all delay and the capacity in the presence of feedback errors are obtained when Q is equal to three.

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7 APPENDIX I

7.1 Expression for $R^{(1)}(f, \lambda)$

It follows from (28) that

$$R^{(1)}(f;\lambda) = (1-\epsilon)\mu e^{-u}. \quad (63)$$

7.2 Expression for $R^{(1)}(g, \lambda)$

$$(1-\epsilon) - R^{(1)}(g;\lambda) = (1-\epsilon)(1-\mu)e^{-\mu}. \quad (64)$$

7.3 Expression for $R^{(1)}(\rho, \lambda)$

Taking the derivative of $\rho(z)$

$$\begin{aligned} \rho^{(1)}(z) &= \left(\frac{Q-1}{2}\right)[L_e^*(\lambda + z/Q) - e^{-z}L_e^*(\lambda)] \\ &= z\left(\frac{Q-1}{2}\right)[\frac{1}{Q}L_e^{*(1)}(\lambda + z/Q) \\ &\quad + e^{-z}L_e^*(\lambda)] \end{aligned} \quad (65)$$

By noting $\lambda + \mu/Q = \mu$ and $\rho^{(1)}(0) = 0$, it follows from (28) that

$$\begin{aligned} R^{(1)}(\rho;\lambda) &= \left(\frac{Q-1}{2}\right)[L_e^*(\mu) - e^{-\mu}L_e^*(\lambda)] \\ &= \mu\left(\frac{Q-1}{2}\right)[\frac{1}{Q}L_e^{*(1)}(\mu) \\ &\quad + e^{-\mu}L_e^*(\lambda)]. \end{aligned} \quad (66)$$

Using (25) and (33), $L_e^*(\mu)$ can be expressed as

$$L_e^*(\mu) = \frac{1}{1-Q}\left\{1 + Q\left[f(\mu) + \frac{\mu}{1-\mu}g(\mu)\right]L_e^*(\lambda)\right\} \quad (67)$$

where f(z) and g(z) are given in (26). Using (18) and (33), $L_e^{*(1)}(\mu)$ can be expressed as

$$\begin{aligned} L_e^{*(1)}(\mu) &= L^{*(1)}(\mu)\left\{1 + (Q-1)\epsilon L_e^*(\lambda) + \lambda\epsilon L_e^{*(1)}(\lambda)\right\} \\ &\quad + \epsilon L_e^{*(1)}(\lambda) \end{aligned} \quad (68)$$

and

$$L^{*(1)} = Q\frac{\mu}{1-\mu}e^{-\mu}L^*(\lambda)\sum_{m=0}^{\infty}Q^{-m}e^{uQ^{-m}}. \quad (69)$$

7.4 Expression for $R(g, \lambda)$

It follows from (28) that

$$R(g,\lambda) = \sum_{m=0}^{\infty}Q^m[g(\lambda_{m+1}) - g(\lambda_m) - \lambda Q^{-m}g^{(1)}(\lambda_m)]. \quad (70)$$

Substitute (26) into (70) yields the following infinite series expression for $R(g;\lambda)$:

$$\begin{aligned} R(g,\lambda) &= (1-\epsilon)\mu e^{-\mu}\sum_{m=0}^{\infty}Q^m\left\{[(1-Q^{-(m+1)})\right. \\ &\quad -\lambda Q^{-m}(1-Q^{-m})]e^{\mu Q^{-m}} \\ &\quad \left. -(1-Q^{-(m+1)})e^{\mu Q^{-(m+1)}}\right\}. \end{aligned} \quad (71)$$

The straight forward calculation of $R(g;\lambda)$ results an numerical unstable summation. By expending the exponential terms in equation (71) into the infinite series and interchanging the order of summation, and simplifying the resulting equation, $R(g;\lambda)$ can be written as

$$\begin{aligned} R(g;\lambda) &= (1-\epsilon)\mu^2 e^{-\mu}\left\{1-Q^{-1}-\mu+\frac{\lambda}{1-Q^{-2}}\right\} \\ &+ (1-\epsilon)\mu e^{-\mu}\sum_{i=2}^{\infty}\frac{\mu^i}{i!}\left\{\frac{1-Q^{-i}}{1-Q^{-(i-1)}}\right. \\ &+ \left.\frac{Q^{-i-1}-Q^{-1}-\lambda}{1-Q^{-i}}+\frac{\lambda}{1-Q^{-(i+1)}}\right\}. \end{aligned} \quad (72)$$

7.5 Expression for $R(f,\lambda)$

To obtain an expression for $R(f;\lambda)$, note that

$$f(z) = g(z) - (1-\epsilon)e^{-z}. \quad (73)$$

Hence,

$$R(f;\lambda) = R(g;\lambda) - (1-\epsilon)R(e^{-z};\lambda). \quad (74)$$

$R(f;\lambda)$ can be written as

$$\begin{aligned} R(f;\lambda) &= R(g;\lambda) - (1-\epsilon)\mu^2 e^{-\mu} + (1-\epsilon)e^{-\mu}\sum_{i=2}^{\infty}\frac{\mu^i}{i!} \\ & \left\{\frac{1-Q^{-i}}{1-Q^{-(i-1)}}-\frac{\lambda}{1-Q^{-i}}\right\}. \end{aligned} \quad (75)$$

7.6 Expression for $R(\rho,\lambda)$

Similar to (70),

$$\begin{aligned} R(\rho;\lambda) &= \sum_{m=1}^{\infty} Q^{m-1}[\rho(\lambda_m)-\rho(\lambda_{m-1}) \\ &\quad -\lambda Q^{-(m-1)}\rho^{(1)}(\lambda_{m-1})]. \end{aligned} \quad (76)$$

Substitute (58) into (76) yields the following infinite series expression for $R(\rho;\lambda)$:

$$\begin{aligned} R(\rho;\lambda) &= \mu e^{-\mu}L_e^*(\lambda)\left(\frac{Q-1}{2}\right)\sum_{m=1}^{\infty}Q^{m-1} \\ & \left\{(1-Q^m)(e^{\mu Q^{-(m-1)}}-e^{\mu Q^{-m}})\right. \\ & \left.-\lambda\, Q^{-(m-1)}(1-Q^{-(m-1)})e^{\mu Q^{-(m-1)}}\right\} \\ & +\mu\left(\frac{Q-1}{2Q}\right)\sum_{m=1}^{\infty}Q^m\{(1-Q^m) \\ & [L_e^*(\lambda_{m+1})-L_e^*(\lambda_m)] \\ & -\lambda\, Q^{-m}(1-Q^{-(m-1)})L_e^{*(1)}(\lambda_m)\Big\}. \end{aligned} \quad (77)$$

The straight forward calculation of $R(\rho;\lambda)$ results an numerical unstable summation. By expending the exponential terms in summation number one of equation (77) into the infinite series, interchanging the order of summation, simplifying the resulting equation, and let the first summation equals to A, A can be written as

$$\begin{aligned} A &= \mu e^{-\mu}L_e^*(\lambda)\left(\frac{Q-1}{2}\right)\left\{\left[\frac{\lambda(1-\mu)}{1-Q^{-1}}-\mu Q^{-1}\right.\right. \\ & \left.+\frac{\lambda}{1-Q^{-2}}\right]+\sum_{i=2}^{\infty}\frac{\mu^i}{i!}\left[\frac{1-Q^{-i}}{1-Q^{-(i-1)}}\right. \\ & -Q^{-i}\frac{1-Q^{-1}}{1-Q^{-i}}-\frac{\lambda}{1-Q^{-i}} \\ & \left.\left.+\frac{\lambda}{1-Q^{-(i+1)}}\right]\right\}. \end{aligned} \quad (78)$$

The sequences $\{L_e^*(\lambda_m)\}$ and $\{L_e^{*(1)}(\lambda_m)\}$ in the second summation of (77) can be generated recursively as follows. Recall that $\lambda+\lambda_m = \lambda_{m+1}$. It then follows by setting $z=\lambda_m$ in (25) for $m \geq 0$,

$$\begin{aligned} L_e^*(\lambda_{m+1}) &= (1/Q)[L_e^*(\lambda_m)-1] \\ &\quad +L_e^*(\lambda)(1-\epsilon) \\ &\quad \left[1+\frac{\mu}{1-\mu}(1-Q^{-m})\right]e^{-\mu}e^{\mu Q^{-m}} \end{aligned} \quad (79)$$

with initial condition $L_e^*(\lambda_0) = L_e^*(0)$. Similarly, first differentiate (25) with respect to z and then set $z=\lambda_m$ in the resulting equation. This yields the following recursion valid for $m \geq 0$:

$$L_e^{*(1)}(\lambda_{m+1}) = L_e^{*(1)}(\lambda_m)+\frac{\mu}{1-\mu}L_e^*(\lambda)Q^{-(m-1)}e^{-\mu}e^{\mu Q^{-m}} \quad (80)$$

with initial condition $L_e^{*(1)}(\lambda_0) = L_e^{*(1)}(0)$. The direct computation of the summation number two in (77) also results in numerical instability since the convergent rate of the first term in the summation number two is faster than the convergent rate of the second term. For large m,

$$\begin{aligned} L_e^{*(1)} &= \frac{L_e^*(\lambda_{m+1})-L_e^*(\lambda_m)}{\lambda_{m+1}-\lambda_m} \\ &= \frac{L_e^*(\lambda_{m+1})-L_e^*(\lambda_m)}{\lambda Q^{-m}} \end{aligned} \quad (81)$$

Then the second summation, B, of (77) can be written as

$$\begin{aligned} B &= \mu\left(\frac{Q-1}{2Q}\right)\sum_{m=1}^{M}Q^m\{(1-Q^m) \\ & [L_e^*(\lambda_{m+1})-L_e^*(\lambda_m)] \\ & -\lambda\, Q^{-m}(1-Q^{-(m-1)})L_e^{*(1)}(\lambda_m)\Big\} \\ & +\mu\left(\frac{Q-1}{2Q}\right)\sum_{m=M}^{\infty}(Q-1) \\ & [L_e^*(\lambda_{m+1})-L_e^*(\lambda_m)]. \end{aligned} \quad (82)$$

where M is larger. And

$$R(\rho;\lambda) = A+B. \quad (83)$$

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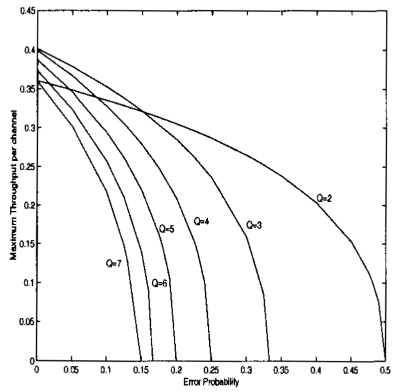

Figure 3: The maximum stable throughput vs. the error probability. Q is equal to the number of partitions or groups.

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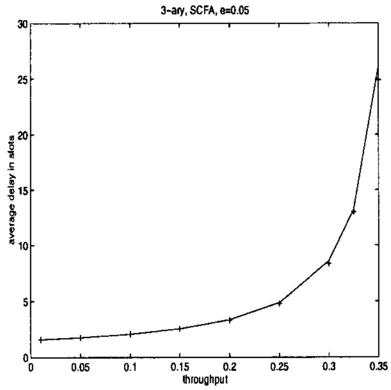

Figure 4: Comparison of simulated and analytical results for Q=3 in the presence of the feedback error $\epsilon = 0.05$.

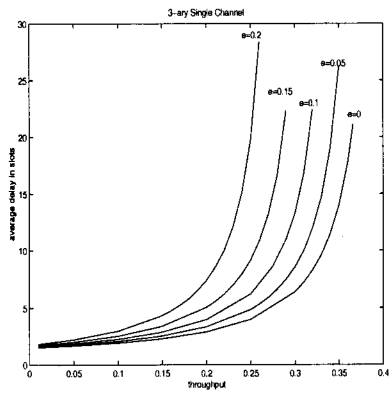

Figure 5: The throughput-delay characteristic for Q=3 in the presence of the feedback error e.

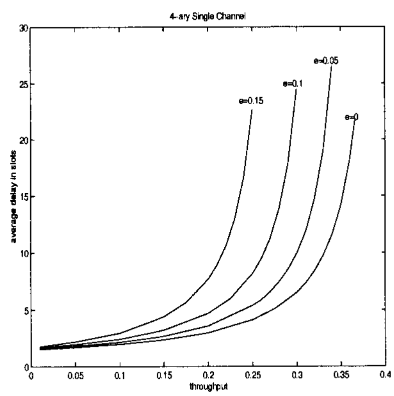

Figure 6: The throughput-delay characteristic for Q=4 in the presence of the feedback error e.